Pulse repetition rate optimization in SLR stations to provide minimum systematic error of ranging

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Abstract

An analytical model of single-photon satellite laser ranging (SLR) is presented, allowing to estimate the systematic error of ranging caused by fluctuations of photon numbers in the return pulse. It is demonstrated that with a sufficiently small number of photoelectrons in the in the pulse the return pulse intensity fluctuations practically do not affect the ranging systematic error value. It is also demonstrated that this error value may be reduced to a given level by reduction of the number of photoelectrons in the pulse and increase of the repetition rate. An estimate of the minimum required pulse repetition rate is presented.

Background

The basic sources of random errors in time-of-flight (TOF) measurements with a low and fluctuating photon number in the return pulse are:

- The finite pulsewidth (defining the uncertainty in the moment of the return signal photon arrival and corresponding photoelectron appearance)
- Fluctuations of the photoelectrons travel time during their avalanche multiplication in the photodetector (transit time jitter).

The RMS return pulsewidth $2\sigma_t$ is defined not by the laser pulsewidth only, but also by the shape, attitude and dimensions of the target retroreflector array (signature effect). Particularly, the duration of return pulses from LAGEOS and GLONASS retroreflector arrays may vary from 100 ps to 400 ps. The RMS value σ_j of transit time jitter when single-photon returns are detected may vary from 10 ps to 50 ps (depending on the photodetector type and operation mode). Therefore, when single photoelectrons are generated by the return pulse, the one-shot ranging precision, with the above conditions, will be no less than 8…40 mm (correspondingly).

The random errors may be somewhat reduced (about two or three times) by increasing the average number of primary photoelectrons in the photodetector pulse, e.g. by increasing the laser pulse energy. However, a systematic ranging error appears thereby, caused by decrease of the mean delay between the optical signal and the first primary photoelectron generation moment, as well as by decrease of the mean detector transit time.

The RMS ranging error may be reduced by averaging of the multiple TOF measurement results; however, the systematic error cannot be reduced by averaging. In some cases, the systematic error may considerably exceed the RMS error.

Elimination of the systematic error may be achieved by using a high-repetition-rate laser transmitter [1] with a low pulse energy, providing generation in the photodetector of much less than one primary photoelectron per transmit pulse.

Estimation of the systematic ranging error

To estimate the systematic error value, it is necessary to analyze the temporal variations of the return pulse arrival time (Figure 1).

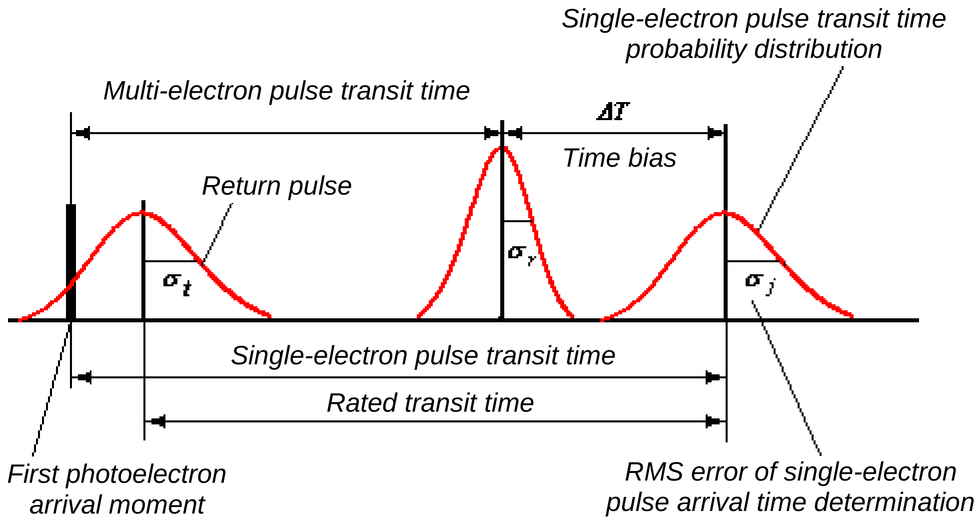

Figure 1. Temporal variations of return pulse arrival time

The process of charge carrier avalanche multiplication is started by the first generated photoelectron within the optical return pulse time limits. The corresponding time moment relative to the optical return pulse centroid varies randomly, and depends on the number of photons in the pulse. The pulse transit time in the photodetector also varies randomly, and also depends on the number of photons (and related photoelectrons) in the pulse. With an increase of the photon (and photoelectron) number, the delay in the photodetector output pulse arrival time is reduced because of the earlier first photoelectron generation as well as because of the detection transit time reduction.

If the return signal intensity does not fluctuate, the probability of $\boldsymbol{n}$ photoelectrons detection in the pulse is defined by the Poisson law:

$$P(n, n_{se}) = \frac{n_{se}^{n}}{n!} \cdot \exp(-n_{se}) \qquad (1)$$

where $\boldsymbol{n_{se}}$ is the average number of photoelectrons generated by the return pulse. In this case, with a Gaussian approximation of the return pulse shape, the probability density of the first photoelectron appearance time distribution may be presented as follows:

$$p_p(t_1, n_{se}) = \frac{n_{se}}{\sqrt{2\pi}} \cdot \frac{\exp(-t_1^2/2)}{1-\exp(-n_{se})} \cdot \exp\left[-\frac{n_{se}}{\sqrt{2\pi}} \int_{-\infty}^{t_1} \exp(-x^2/2)dx\right] \qquad (2)$$

where $\boldsymbol{t_1}$ is the first photoelectron appearance time normed to the return pulse half-width $\boldsymbol{\sigma_t}$. In Figure 2, the $\boldsymbol{p_p(t_1,n_{se})}$ function shape is shown for different values of $\boldsymbol{n_{se}}$.

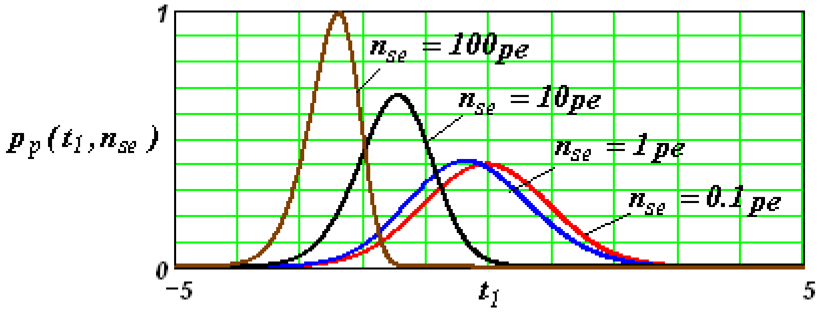

Figure 2. Probability density of the first photoelectron appearance time for non-fluctuating return signals

With n_{se}<<*1*, the curve of probability density of the first photoelectron appearance time has the same shape as the optical return pulse, and the appearance time has a zero mathematical expectation, i.e. the mean time of the first photoelectron appearance corresponds to the return pulse centroid. With an increase of n_{se}, the mean first photoelectron appearance time delay is reduced, thus causing a systematical ranging error which may be estimated from the expression:

$$\Delta t_{tr} = \int_{-\infty}^{\infty} t_1 \cdot p_p(t_1, n_{se}) \cdot dt_1 \qquad (3)$$

where Δt_{tr} is the systematic ranging error normed to the return pulse half-width. If the return signal intensity fluctuates, the probability density of the first photoelectron appearance time $p_b(t_1,n_{se})$ may be determinated from the expression:

$$p_b(t_1, n_{se}) = \int_0^{\infty} p_p(t_1, n) \cdot p(n) \cdot dn \qquad (4)$$

where $p(n)$ is the return pulse intensity probability density.

In a common case, the optical return pulse intensity fluctuations are caused by the atmosphere turbulence at the upward and downward laser beam propagation path, as well as by interference effects arising when the laser pulse is reflected by multiple cube corner reflectors in the retroreflector array [2]. There are sufficient theoretical and experimental data providing the possibility of adequate numerical estimation of the resulting probability density taking into account the turbulent atmosphere effects as well as the specle effects caused by reflection from multiple cube corners. However, for analytical estimations it would make sense to use an approximation of the resulting probability density in the form of an exponential distribution:

$$p(n) = \frac{1}{n_{se}} \cdot \exp(-\frac{n}{n_{se}}) \qquad (5)$$

In this case, the probability of n photoelectrons detection in the return signal pulse is defined by the Bose-Einstein distribution:

$$P(n, n_{se}) = \frac{1}{1+n_{se}} \cdot \left(\frac{n_{se}}{1+n_{se}}\right)^n \qquad (6)$$

and the probability density of the first photoelectron appearance time calculated in accordance with the equation (4) has a form shown in Figure 3:

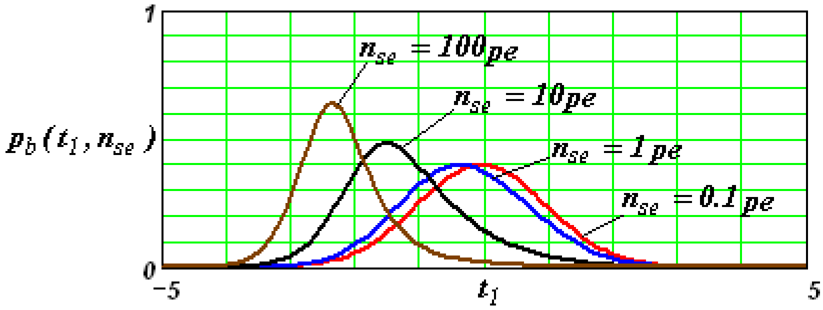

Figure 3. Probability density of the first photoelectron appearance time for fluctuating return signal pulses

From the expressions (1-6) one may calculate the systematic ranging error ΔT_{tr} caused by change in the first photoelectron appearance time on the average number of photoelectron in the pulse. The dependence is shown in Figure 4:

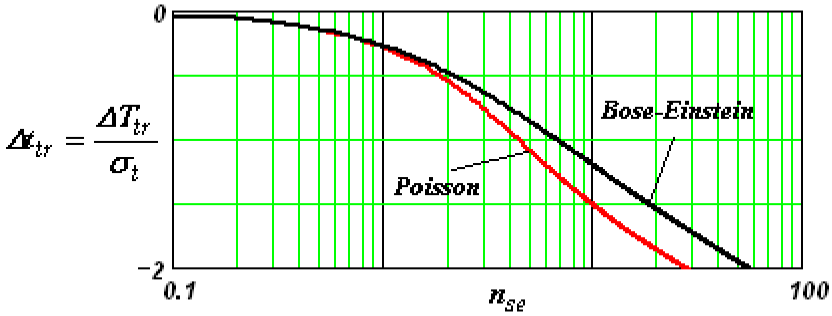

Figure 4. Estimated time bias caused by first photoelectron appearance time variations

One may see from the curves that when the average number of photoelectrons in the pulse is more than 6, the systematic error of measurements (time bias) is always more than σ_t. The σ_t behavior at low n_{se} values will be analyzed below.

For a quantitative estimation of the photoelectrons number in the pulse effect on the transit time, it is worth while to use the results presented in [3]. In this publication, a model is proposed of the avalanche buildup process in the photodetector. In accordance with this model, the transit time is interpreted as the mean time needed for duplication of the number of charge carriers multiplied by the number of duplications needed for exceeding the threshold value of the comparator in the SLR station receiver. In this case, the transit time bias dependence on the average number of photoelectrons in the pulse is defined by the expression:

$$\Delta T_r = \frac{\sigma_j}{\ln 2} \cdot \left[\frac{\sum_{n=1}^{\infty} P(n, n_{se}) \cdot \ln(n)}{\sum_{n=1}^{\infty} P(n, n_{se})} \right] \qquad (7)$$

where σ_j is the RMS deviation of the transit time value for the single-electron pulse. In Figure 5, the dependence is shown, calculated in accordance with (7) for fluctuating and non-fluctuating return signals:

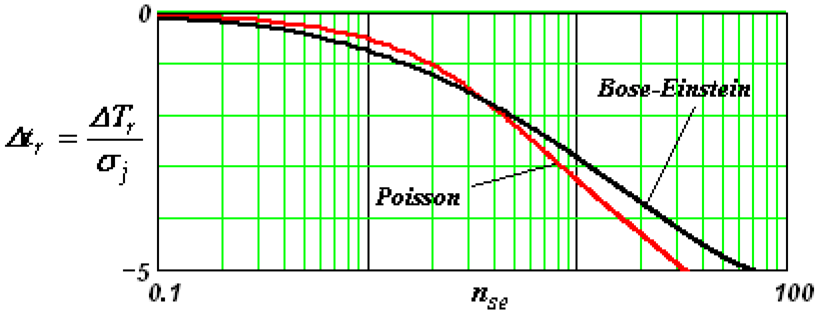

Figure 5. Estimated time bias caused by pulse transit time variations in the photodetector

The systematic ranging error here dominates over the σ_j value already when the average number of photoelectrons in the return signal is more than 2.

Estimation of the required average number of photoelectrons in the return signal pulse

The resulting bias in the return signal pulse arrival time, in accordance with the relationship (3) and (7), may be presented as:

$$\Delta T_{bias} = \sigma_t \cdot \Delta t_{tr} + \sigma_j \cdot \Delta t_r \qquad (8)$$

To retain resulting bias below a certain level, the average number of photoelectrons in the pulse should be small enough. In Tables 1 and 2, permissible values are presented of the average photoelectrons number in the pulse, calculated in accordance with (8) for permissible values of range estimation bias 0.3 mm and 0.6 mm (correspondingly).

Table 1. Permissible bias of estimation: ΔT_{bias} = 2 ps, σ_j = 20 ps

Pulsewidth		**50 ps**	**120 ps**	**240 ps**	**480 ps**
n_{se}	**Poisson**	0.123	0.074	0.041	0.017
	Bose-Einstein	0.077	0.053	0.033	0.015

Table 2. Permissible bias of estimation: ΔT_{bias} = 4 ps, σ_j = 20 ps

Pulsewidth		**50 ps**	**120 ps**	**240 ps**	**480 ps**
n_{se}	Poisson	0.250	0.157	0.094	0.047
	Bose-Einstein	0.163	0.116	0.077	0.042

The following conclusions may be drawn from the calculation results:

- The effect of intensity fluctuations on the ranging systematic error decreases with decrease of the average number of photoelectrons in the return signal pulse.

- At $n_{se} \le 0.05$, the ranging systematic error does not exceed 0.6 mm for return pulsewidth less than 400 ps.
- At $n_{se} \le 0.02$, the ranging systematic error does not exceed 0.3 mm for return pulsewidth less than 400 ps.

Estimation of the required pulse repetition rate

To provide the SLR station operation at a sufficiently small n_{se} level, it is necessary to meet the requirement:

$$F \ge \frac{f_{se}}{n_{se}} \tag{9}$$

where F is the laser pulse repetition rate, and f_{se} is the mean value of return signal photoelectron generation frequency in the SLR station receiver (i.e., the average number of signal-initiated photoelectrons per second).

The mean frequency of signal photoelectron appearance depends on the average laser power and may be determined from the equation:

$$f_{se} = \frac{\eta}{h\nu} \cdot \frac{4P_{av}}{\pi\theta_t^2} \cdot \sigma \cdot \frac{A_r}{4\pi R^4} \cdot \tau_{opt} \cdot \tau_{atm} \tag{10}$$

where η is the photodetector quantum efficiency, $h\nu$ is the quantum energy, P_{av} is the laser average power, θ_t is the laser transmitter output beamwidth, σ is the equivalent retroreflector array cross-section, A_r is the receive telescope aperture area, R is the target range, τ_{opt} is the optical system transmission, and τ_{atm} is the two-way atmosphere transmission.

For a compact SLR station with an average laser power of about 1 W (e.g. for the SAZHEN-TM SLR station) the f_{se} value is about 10 sec^{-1} with a GLONASS satellite being the target, and about 100 sec^{-1} with a LAGEOS. For SLR stations having the same average power but a 0.5 m-diameter aperture, the f_{se} value is usually between 200 and 2000 sec^{-1}. Hence, to provide $n_{se} \le 0.5$ a corresponding pulse repetition rate of 4 kHz to 40 kHz is necessary.

The requirement for the pulse repetition rate value may be mitigated by reducing the f_{se} value, e.g. by an increase of the laser beamwidth; however, the SLR station productivity will be thereby less, because the f_{se} value is the maximum return rate and hence a decrease of this value will increase the time necessary to obtain a sufficient number of individual range measurements.

Thus, to eliminate the ranging systematic errors with a simultaneous increase or retaining of the station productivity, it is desirable to use laser pulse repetition rates of several tens of kHz.

References

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